计算概论A—实验班

函数式程序设计

Functional Programming

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第3章： 类型与类簇

type and type class

What is a Type?

A type is a collection of related values

For example, in Haskell
the basic type `Bool`, contains two logical values
`True`, and `False`

Type Errors / 类型错误

Applying a function to one or more arguments of the wrong type is called a type error

1 is a number and `False` is a logical value
but + requires two numbers

nrutas — ghc-9.4.2 -B/Users/nrutas/.ghcup/ghc/9.4.2/lib/ghc-9.4.2/lib --interactive — 62×8

```
ghci> 1 + False

<interactive>:1:1: error:
    • No instance for (Num Bool) arising from the literal ‘1’
    • In the first argument of ‘(+)’, namely ‘1’
      In the expression: 1 + False
      In an equation for ‘it’: it = 1 + False
ghci>
```

Types in Haskell

- If evaluating an expression e would produce a value of type T, then e has type T, written

  e :: T

- Every well formed expression has a type, which can be automatically calculated at compile time using a process called type inference.

$$\frac{f :: A \rightarrow B, \; e :: A}{f\,e :: B}$$

Types in Haskell

- All type errors are found at compile time, which makes programs safer and faster by removing the need for type checks at run time
- In GHCi, the `:type` command calculates the type of an expression, without evaluating it

nrutas — ghc-9.4.2 -B/...

```
ghci> not False
True
ghci> :type not False
not False :: Bool
ghci>
```

Basic Types in Haskell

`Bool`	▸ logical values: `True False` ▸ exported by `Prelude`
`Char`	▸ an enumeration whose values represent Unicode code points (i.e. characters, see http://www.unicode.org/ for details) ▸ exported by Prelude
`String`	▸ definition: `type String = [char]` ▸ exported by Prelude

Basic Types in Haskell

`Int`	▸ fix-precision integer numbers. GHC: [-2^63, 2^63-1] ▸ exported by `Prelude`
`Integer`	▸ arbitrary-precision integer numbers ▸ exported by `Prelude`
`Word`	▸ fix-precision unsigned integer numbers ▸ the same size with `Int` ▸ exported by `Prelude`
`Natural`	▸ arbitrary-precision unsigned integer numbers ▸ exported by Numeric.Natural (a module in the base package)

Basic Types in Haskell

Float	‣ single-precision floating-point numbers ‣ exported by Prelude
Double	‣ double-precision floating-point numbers ‣ exported by Prelude

nrutas — ghc-9.4.2 -B/U...

```
ghci> sqrt 2 :: Float
1.4142135
ghci> sqrt 2 :: Double
1.4142135623730951
ghci>
```

List Types

A list is a sequence of values of the same type

nrutas — ghc-9.4.2 -B/Users/nrutas/.ghcu...

```
ghci> :type [False, True, False]
[False, True, False] :: [Bool]
ghci> :type ['a', 'b', 'c', 'd']
['a', 'b', 'c', 'd'] :: [Char]
ghci>
```

Given a type T:

[T] is the type of of lists with elements of type T

List Types

Note 1	▸ The type of a list says nothing about the list's length
Note 2	▸ The type of the elements is unrestricted ▸ For example, we can have lists of lists

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```
ghci>
ghci>
ghci> :type [['a'], ['b', 'c'], []]
[['a'], ['b', 'c'], []] :: [[Char]]
ghci>
```

Function Types

A function is a mapping from values of one type to values of another type

```
-- | Boolean \"not\"
not                        :: Bool -> Bool
not True                   =  False
not False                  =  True
```

Given two types X and Y:

X -> Y is the type of functions

that map values of X to values of Y

Function Types

Note 2	▸ The argument and result types are unrestricted ▸ For example, functions with multiple arguments or results are possible using lists or tuples

```
add :: (Int,Int) -> Int
add (x,y) = x+y

zeroto :: Int -> [Int]
zeroto n = [0..n]
```

Curried Functions

Functions with multiple arguments are also possible by returning functions as results

```
add :: (Int,Int) -> Int
add (x,y) = x+y
```

```
add' :: Int -> Int -> Int
add' x y = x + y
```

- add' takes an integer x and returns a function add' x
- add' x takes an integer y and returns the result x+y

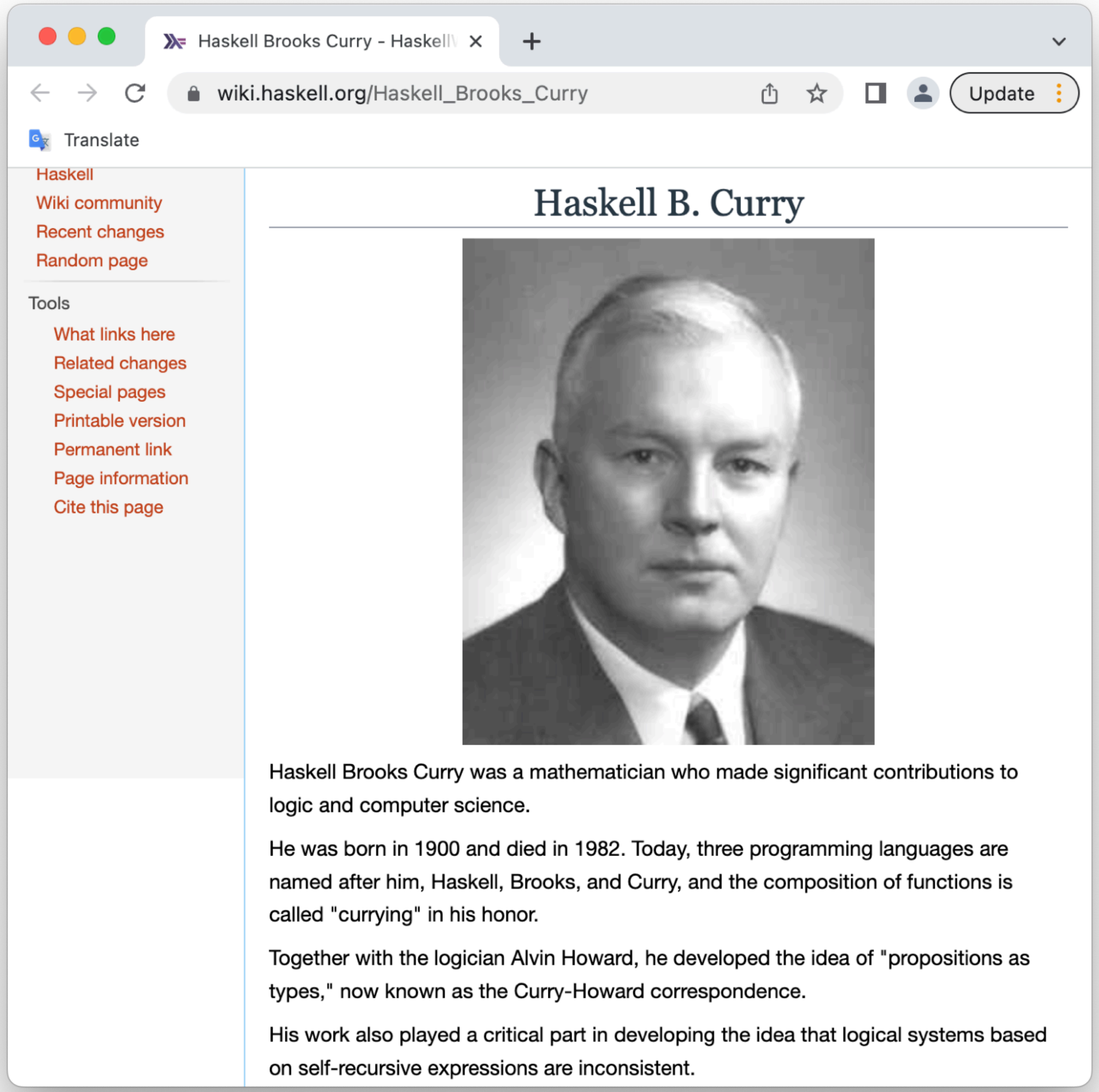
Curried Functions

```haskell
add :: (Int,Int) -> Int
add (x,y) = x+y
```

```haskell
add' :: Int -> Int -> Int
add' x y = x + y
```

- add and add' produce the same final result,
- but add takes its two arguments at the same time, whereas add' takes them one at a time

Functions that take their arguments one at a time are called curried functions, celebrating the work of Haskell Curry on such functions.



Curried Functions

- Functions with more than two arguments can be curried by returning nested functions.

```
mult :: Int -> Int -> Int -> Int
mult x y z = x * y * z
```

- mult takes an integer x and returns a function mult x,
  - which in turn takes an integer y and returns a function mult x y,
    - which finally takes an integer z and returns the result x*y*z.

Why is Currying Useful?

- Curried functions are more flexible than functions on tuples.
- Useful functions can often be made by partially applying a curried function.
- For example:

```
add' 1 :: Int -> Int
take 5 :: [Int] -> [Int]
drop 5 :: [Int] -> [Int]
```

Currying Conventions

- The arrow -> associates to the right

`Int -> Int -> Int -> Int`	<=>	`Int -> (Int -> (Int -> Int))`

- As a consequence, it is then natural for function application to associate to the left.

`mult x y z`	<=>	`((mult x) y) z`

Unless tupling is explicitly required,
all functions in Haskell are normally defined in curried form.

Polymorphic Functions

A function is called polymorphic ("of many forms") if its type contains one or more type variables

```
length :: [a] -> Int
```

- For any type a, length takes a list of values of type a and returns an integer

Polymorphic Functions

- Type variables can be instantiated to different types in different circumstances:

nrutas — ghc-9.4.2 -B/Users/nrutas/.gh...

```
ghci>
ghci> length [True, False, True]
3
ghci> length [0, 1, 1, 2]
4
ghci>
```

a = Bool

a = Int

- Type variables must begin with a lower-case letter, and are usually named a, b, c, etc.

Polymorphic Functions

- Many of the functions defined in the standard prelude are polymorphic. For example:

```
fst :: (a, b) -> a
```

Extract the first component of a pair.

```
snd :: (a, b) -> b
```

Extract the second component of a pair.

```
curry :: ((a, b) -> c) -> a -> b -> c
```

`curry` converts an uncurried function to a curried function.

Examples

```
>>> curry fst 1 2
1
```

```
head :: [a] -> a
```

$\mathcal{O}(1)$. Extract the first element of a list, which must be non-empty.

```
>>> head [1, 2, 3]
1
>>> head [1..]
1
>>> head []
*** Exception: Prelude.head: empty list
```

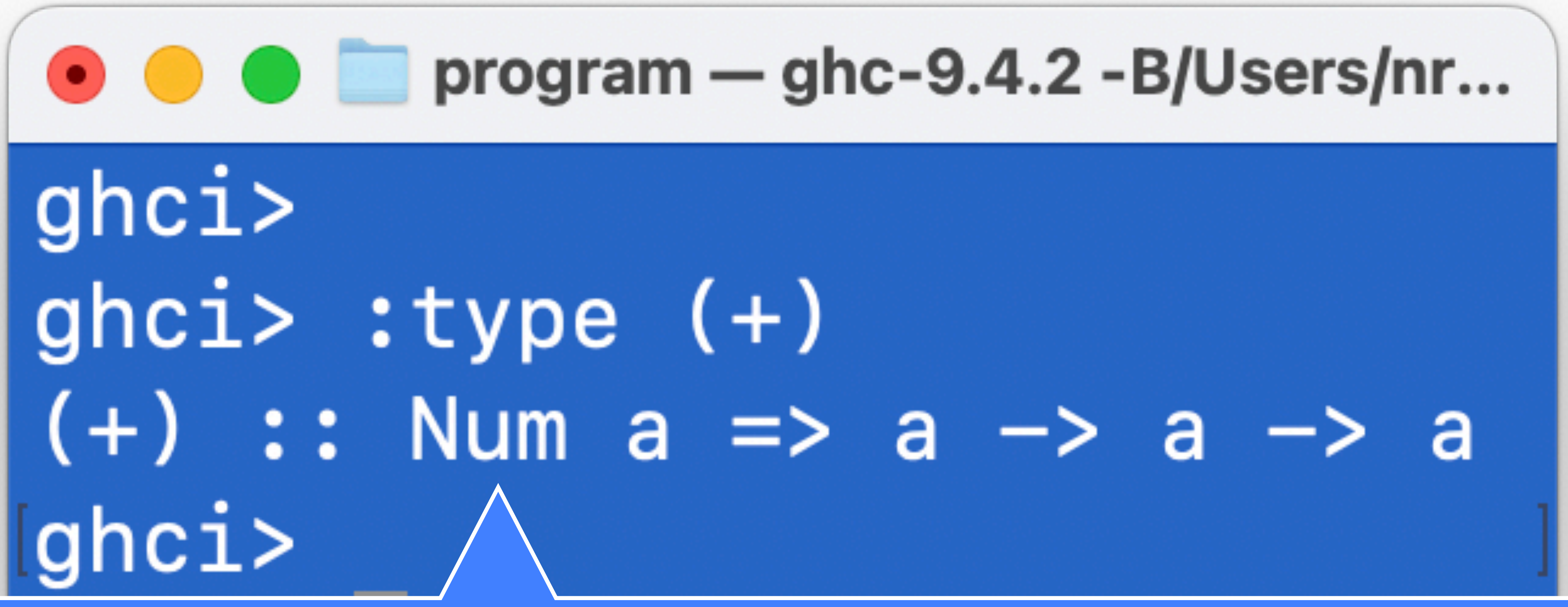
```
last :: [a] -> a
```

$\mathcal{O}(n)$. Extract the last element of a list, which must be finite and non-empty.

```
>>> last [1, 2, 3]
3
>>> last [1..]
* Hangs forever *
>>> last []
*** Exception: Prelude.last: empty list
```

Overloaded Functions

A polymorphic function is called overloaded if its type contains one or more *type class* constraints



For any type a that is an instance of type class Num,
(+) takes two values of type a and returns a value of type a.

Overloaded Functions

- Constrained type variables can be instantiated to any types that satisfy the constraints:

program — ghc-9.4.2 -B/Users/nrutas/.ghcup/ghc/9.4.2/lib/ghc-9.4.2/lib --interactive —...

```
ghci>
ghci> 1 + 2
3
ghci> 1.0 + 2.0
3.0
ghci> 'a' + 'c'

<interactive>:14:5: error:
    • No instance for (Num Char) arising from a use of ‘+’
    • In the expression: 'a' + 'c'
      In an equation for ‘it’: it = 'a' + 'c'
ghci>
```

char is not an instance of type class Num

Type Class

- Prelude exports many type classes, for example:
  - `Eq`: Equality types
  - `Ord`: Ordered types
  - `Num`: Numeric types
- These type classes appear in many types of functions

program — ghc-9.4.2 -B/Users/nrutas/.gh...

```
ghci>
ghci> :type (==)
(==) :: Eq a => a -> a -> Bool
ghci>
ghci> :type (<)
(<) :: Ord a => a -> a -> Bool
ghci>
ghci> :type (+)
(+) :: Num a => a -> a -> a
ghci>
```

Type Class: Eq

```haskell
class Eq a where
    (==), (/=) :: a -> a -> Bool
    x /= y     = not (x == y)
    x == y     = not (x /= y)
```

- 左侧是定义Eq的源代码
- 但是，有很多信息没有表现出来

✳ The Eq class defines equality (==) and inequality (/=).

✳ All basic datatypes exported by Prelude are instances of Eq.

✳ Eq may be derived for any datatype whose constituents are also instances of Eq.

Type Class: Eq

- The Haskell Report defines no laws for Eq.
- However, instances are encouraged to follow these properties:

Reflexivity

`x == x` = `True`

Symmetry

`x == y` = `y == x`

Transitivity

if `x == y && y == z` = `True`, then `x == z` = `True`

Extensionality

if `x == y` = `True` and `f` is a function whose return type is an instance of `Eq`, then `f x == f y` = `True`

Negation

`x /= y` = `not (x == y)`

Type Class: Eq

Minimal complete definition

```
(==) | (/=)
```

如果你想将类型T声明为Eq的实例
只需提供(==)和(/=)两者之一在T上的实现

Methods

```
(==) :: a -> a -> Bool    | infix 4 |

(/=) :: a -> a -> Bool    | infix 4 |
```

Type Class: Ord

```haskell
class  (Eq a) => Ord a  where
    compare              :: a -> a -> Ordering
    (<), (<=), (>), (>=) :: a -> a -> Bool
    max, min             :: a -> a -> a

    compare x y = if x == y then EQ
                  else if x <= y then LT
                  else GT

    x <  y = case compare x y of { LT -> True;  _ -> False }
    x <= y = case compare x y of { GT -> False; _ -> True }
    x >  y = case compare x y of { GT -> True;  _ -> False }
    x >= y = case compare x y of { LT -> False; _ -> True }

    -- These two default methods use '<=' rather than 'compare'
    -- because the latter is often more expensive
    max x y = if x <= y then y else x
    min x y = if x <= y then x else y
```

以下是类型Ordering的定义

```haskell
data Ordering = LT | EQ | GT
```

Type Class: Ord

- Ord, as defined by the Haskell report, implements a total order and has the following properties:

Comparability

`x <= y || y <= x` = `True`

Transitivity

if `x <= y && y <= z` = `True`, then `x <= z` = `True`

Reflexivity

`x <= x` = `True`

Antisymmetry

if `x <= y && y <= x` = `True`, then `x == y` = `True`

The following operator interactions are expected to hold:

1. `x >= y` = `y <= x`
2. `x < y` = `x <= y && x /= y`
3. `x > y` = `y < x`
4. `x < y` = `compare x y == LT`
5. `x > y` = `compare x y == GT`
6. `x == y` = `compare x y == EQ`
7. `min x y == if x <= y then x else y` = `True`
8. `max x y == if x >= y then x else y` = `True`

Minimal complete definition

```
compare | (<=)
```

如果你想将类型T声明为Ord的实例
只需提供compare和(<=)两者之一在T上的实现

Methods

```
compare :: a -> a -> Ordering

(<) :: a -> a -> Bool        infix 4

(<=) :: a -> a -> Bool       infix 4

(>) :: a -> a -> Bool        infix 4

(>=) :: a -> a -> Bool       infix 4

max :: a -> a -> a

min :: a -> a -> a
```

Type Class: Num

```haskell
class  Num a  where
    {-# MINIMAL (+), (*), abs, signum, fromInteger, (negate | (-)) #-}

    (+), (-), (*)       :: a -> a -> a
    -- Unary negation.
    negate              :: a -> a
    -- Absolute value.
    abs                 :: a -> a
    -- Sign of a number.
    signum              :: a -> a
    -- Conversion from an Integer.
    fromInteger         :: Integer -> a

    x - y               = x + negate y
    negate x            = 0 - x
```

Type Class: Num

- The Haskell Report defines no laws for Num.
- However, (+) and (*) are customarily expected to define a ring and have the following properties:

Associativity of (+)

`(x + y) + z = x + (y + z)`

Commutativity of (+)

`x + y = y + x`

`fromInteger` 0 is the additive identity

`x + fromInteger 0 = x`

`negate` gives the additive inverse

`x + negate x = fromInteger 0`

Associativity of (*)

`(x * y) * z = x * (y * z)`

`fromInteger` 1 is the multiplicative identity

`x * fromInteger 1 = x` and `fromInteger 1 * x = x`

Distributivity of (*) with respect to (+)

`a * (b + c) = (a * b) + (a * c)` and `(b + c) * a = (b * a) + (c * a)`

Minimal complete definition

```
(+), (*), abs, signum, fromInteger, (negate | (-))
```

Methods

```
(+) :: a -> a -> a    infixl 6
```
Source

```
(-) :: a -> a -> a    infixl 6
```
Source

```
(*) :: a -> a -> a    infixl 7
```
Source

```
negate :: a -> a
```
Source

Unary negation.

```
abs :: a -> a
```
Source

Absolute value.

```
signum :: a -> a
```
Source

Sign of a number. The functions `abs` and `signum` should satisfy the law:

```
abs x * signum x == x
```

For real numbers, the `signum` is either -1 (negative), 0 (zero) or 1 (positive).

```
fromInteger :: Integer -> a
```
Source

作业

作业

3-1 What are the types of the following values?

```
['a', 'b', 'c']

('a', 'b', 'c')

[(False, '0'), (True, '1')]

([False, True], ['0', '1'])

[tail, init, reverse]
```

作业

3-2 What are the types of the following functions?

```
second xs = head (tail xs)

swap (x, y) = (y, x)

pair x y = (x, y)

double x = x * 2

palindrome xs = reverse xs == xs

twice f x = f (f x)
```

作业

3-3 阅读教科书，用例子（在ghci上运行）展示Int与Integer的区别以及show和read的用法

3-4 阅读教科书以及Prelude模块的相关文档，理解 Integral 和 Fractional 两个Type Class中定义的函数和运算符，用例子（在ghci上运行）展示每一个函数/运算符的用法

第3章： 类型与类簇
type and type class

就到这里吧